

ATTACHMENT F

Spatial Variability of $PM_{2.5}$ and Other Pollutants

In this attachment the spatial variability of pollutants is characterized by calculating the linear correlation coefficients for each possible pair of sites and plotting these as a function of distance. The data for pollutants other than $PM_{2.5}$ were extracted from AIRS on July 6, 2000, while the $PM_{2.5}$ data were extracted on July 12, 2000.¹ To make the problem tractable, several limitations are placed on AIRS data to screen data from certain pairs of sites.

- Only site pairs within 100 km of each other are considered
- Only sites that have at least 10 data pairs are used
- Only 1999 data are used
- No adjustments are made for site pairs in different times zones

Methods

Computing the distance between two points on the earth given latitude and longitude relies on two relationships: the relationship between spherical coordinates and Euclidian points in 3-space and the two ways of computing a dot product. Latitude is the angle measured from the equator to the center of the earth to the point of interest. Longitude is angle measured from the zero meridian which runs through Greenwich, England, to the center of the earth to the meridian running through the point of interest as measured along the equator. Representing this in 3-dimensional Euclidean space, we have a point, p , on the earth with latitude, α , and longitude, β . Looking at Figure F-1, we can see the relationship between latitude and longitude in

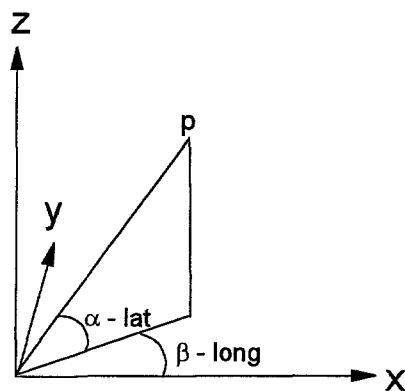


Figure F-1. Latitude and Longitude in Euclidian 3-Space.

spherical coordinates and three dimensional vectors describing the point p :

¹ See Attachment A for a detailed description of the PM data used in these analyses.

$$x = r \cdot \cos(\alpha) \cdot \cos(\beta)$$

$$y = r \cdot \cos(\alpha) \cdot \sin(\beta)$$

$$z = r \cdot \sin(\alpha)$$

where r is the radius of the earth.

Now consider the dot product between the vectors for two different points , p_1 and p_2 . In 3-space the dot product is $\vec{p}_1' \cdot \vec{p}_2$, which is $x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2$ in Euclidian coordinates and is also $r^2 \cos(\theta)$, where θ is the angle between the two vectors representing the two points on the earth. One can find θ by setting the two forms of the dot product equal to each other:

$$x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2 = r^2 \cos(\theta)$$

which by substitution is :

$$\begin{aligned} & r \cos(\alpha_1) \cos(\beta_1) \cdot r \cos(\alpha_2) \cos(\beta_2) + \\ & r \cos(\alpha_1) \sin(\beta_1) \cdot r \cos(\alpha_2) \sin(\beta_2) + \\ & r \sin(\alpha_1) \sin(\alpha_2) = r^2 \cos(\theta) \end{aligned}$$

$$\begin{aligned} & \cos(\alpha_1) \cos(\beta_1) \cos(\alpha_2) \cos(\beta_2) + \\ & \cos(\alpha_1) \sin(\beta_1) \cos(\alpha_2) \sin(\beta_2) + \\ & \sin(\alpha_1) \sin(\alpha_2) = \cos(\theta) \end{aligned}$$

Now we can find the angle θ as:

$$\begin{aligned} \theta = & A \cos[\cos(\alpha_1) \cos(\beta_1) \cos(\alpha_2) \cos(\beta_2) + \\ & \cos(\alpha_1) \sin(\beta_1) \cos(\alpha_2) \sin(\beta_2) + \\ & \sin(\alpha_1) \sin(\alpha_2)] \end{aligned}$$

If we measure all the angles in radians, then the distance along the great circle between the two points is simply $r \cdot \theta$ where r is the radius of the earth. Therefore, the distance between the points, d , is:

$$d = r \bullet A \cos[\cos(\alpha_1) \cos(\beta_1) \cos(\alpha_2) \cos(\beta_2) + \cos(\alpha_1) \sin(\beta_1) \cos(\alpha_2) \sin(\beta_2) + \sin(\alpha_1) \sin(\alpha_2)]$$

However, the earth is not a perfect sphere but somewhat flattened. The polar radius is 6356.912 km while the equatorial radius is 6378.388 km, a difference of 21.476 km. The radius used at a given latitude is the difference, d , times a function that is 0 at ninety degrees, or at the poles, and 1 at 0 degrees, or at the equator. This suggests the use of a cosine function. So the radius is:

$$r = r_p \bullet d \bullet \cos(lat).$$

Finally, we use the average of this when using two latitudes to compute distance.

To compute the correlation we use the standard formula for computing the estimate of the Pearson Product moment correlation coefficient given the paired values as x and y :

$$r = \frac{\sum_{i=1}^n x_i y_i - \frac{\sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n}}{\sqrt{\left(\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right) \left(\sum_{i=1}^n y_i^2 - \frac{\left(\sum_{i=1}^n y_i \right)^2}{n} \right)}}$$

Discussion

Figure F-1 plots the between-site correlation of 24-hour average $PM_{2.5}$ concentrations against the distance between the sites. Figure F-1 supports the notion that $PM_{2.5}$ is a macro-scale or regional pollutant. Looking at Figure F-1, we generally see that the correlation remains very high out to 100 km. The figure even suggests that this would continue beyond 100 km since the correlations do not decrease or decrease very little with any general trend out to this distance. There are many points that do not show this relationship; however, the vast majority of site pairs

do show this relationship. Looking at Figure F-2, we see the same type of plot for $PM_{(10-2.5)}$.² There are not as many sites and the picture we see is different. There is less cohesion in the $PM_{(10-2.5)}$ data, which suggests that $PM_{(10-2.5)}$ is not as regional in nature as $PM_{2.5}$.

It is interesting to compare the spatial variability of $PM_{2.5}$ and $PM_{(10-2.5)}$ to that of other pollutants, such as ozone, carbon monoxide (CO), sulfur dioxide (SO_2), and nitrogen dioxide (NO_2). Since these pollutants are routinely measured on an hourly basis it is possible to calculate hourly correlations. In addition, daily maximums and daily means are also used to produce three plots for each pollutant.

In Figure F-3, the general trend for hourly ozone measurements is a very strong correlation out to 100 km, however, there is a more pronounced downward slope to these correlations than those for $PM_{2.5}$. This indicates that the ozone hourly data begin to disagree at an increased rate with distance when compared to $PM_{2.5}$ data. The daily mean and maximum data for ozone are shown in Figures F-4 and F-5. These statistics create a picture that is very similar to the hourly data in Figure F-3.

CO shows an entirely different story. In Figures F-6, F-7, and F-8 we see that the few high correlations that exist fall off very quickly with distance (less so for the daily mean in Figure F-7). The lack of a general grouping with a trend indicates that CO is not a regional phenomenon, and may be a micro-scale pollutant. SO_2 (Figures F-9, F-10, and F-11) and NO_2 (Figures F-12, F-13, and F-14) have similar patterns indicating the same scale of the pollution problem.

² See Attachment A or Attachment D for a discussion of the methods used to develop $PM_{(10-2.5)}$ data.

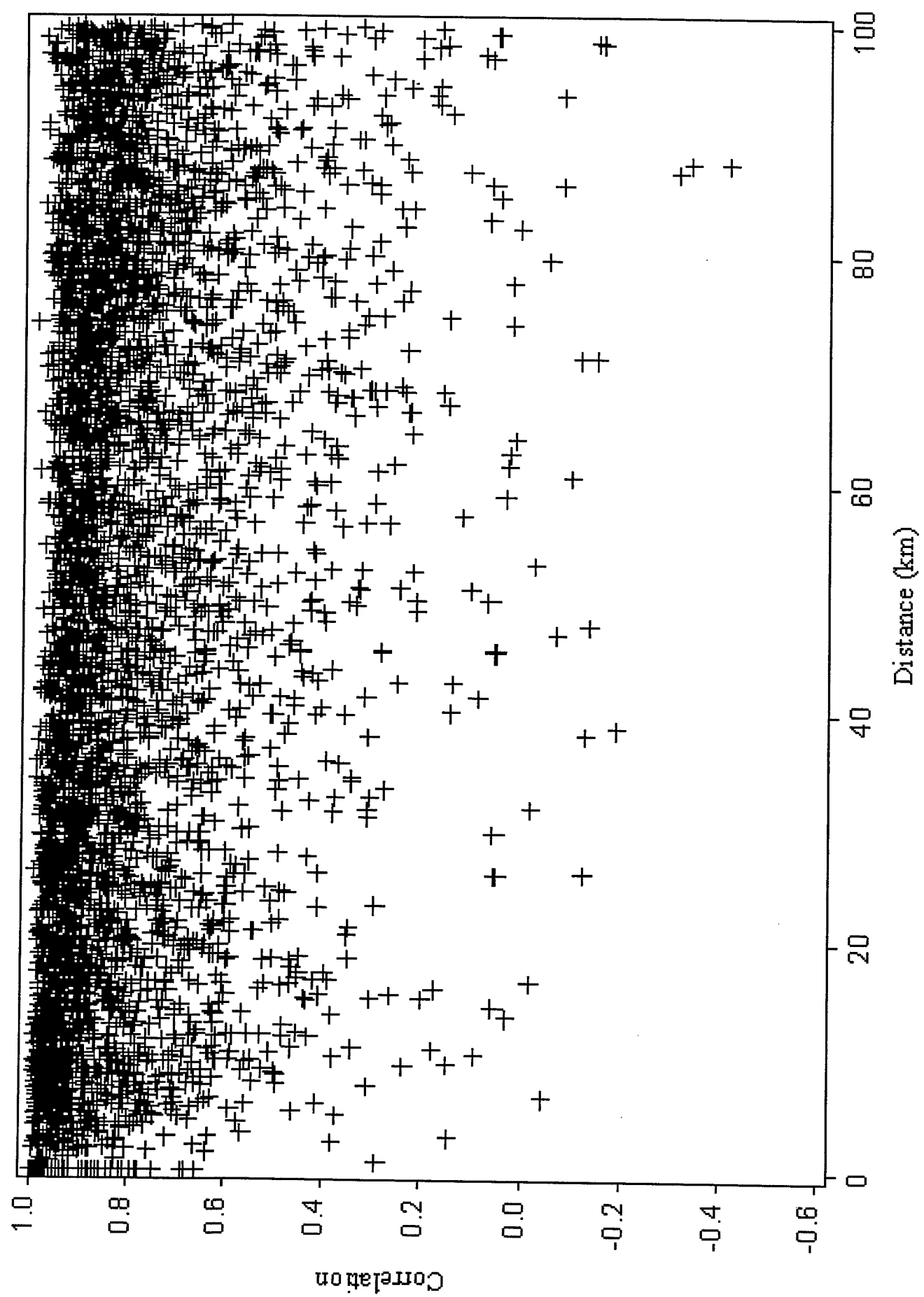


Figure F-1. PM_{4.5} Correlations vs. Distance

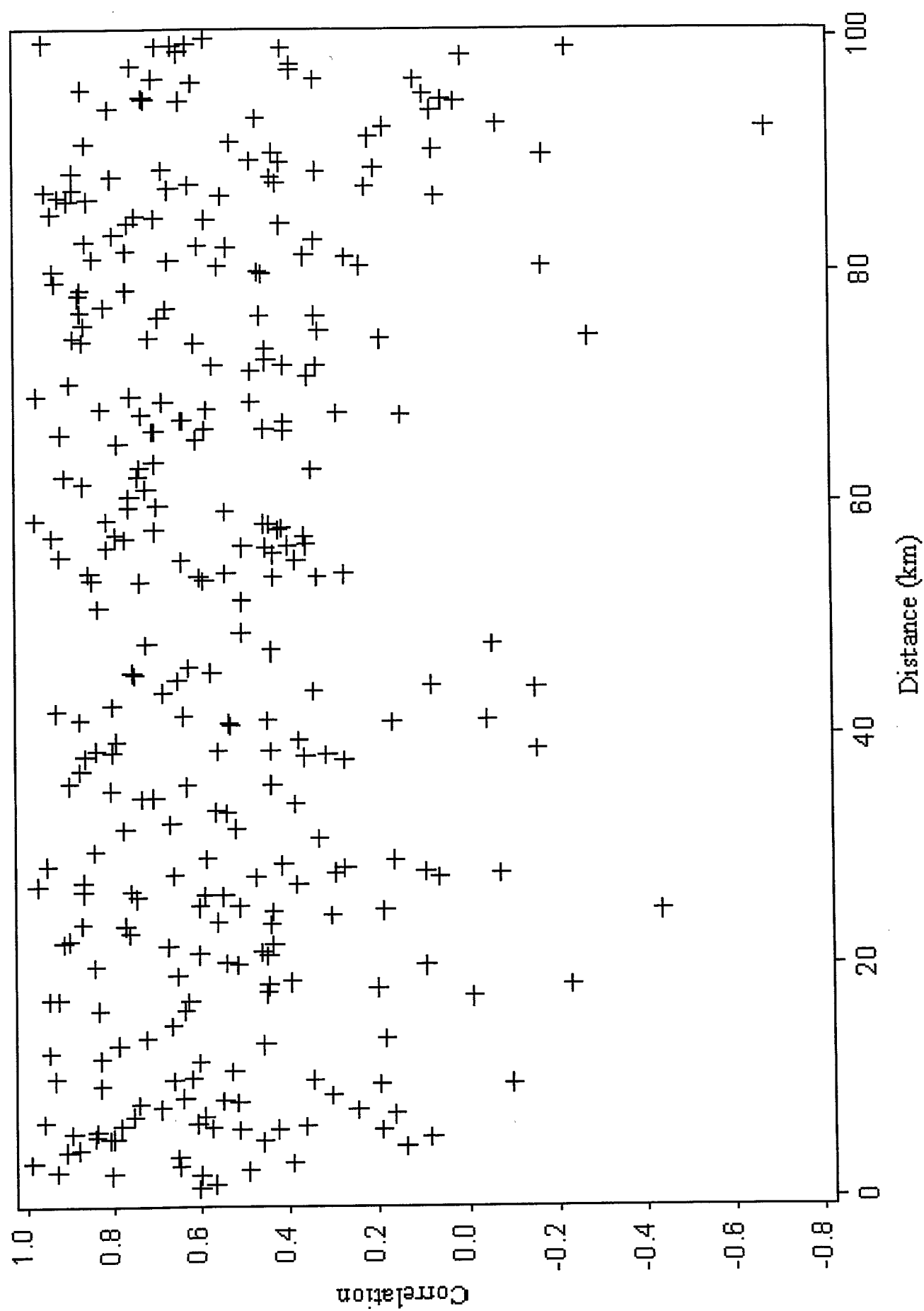


Figure F-2. PM₁₀ Correlations vs distance

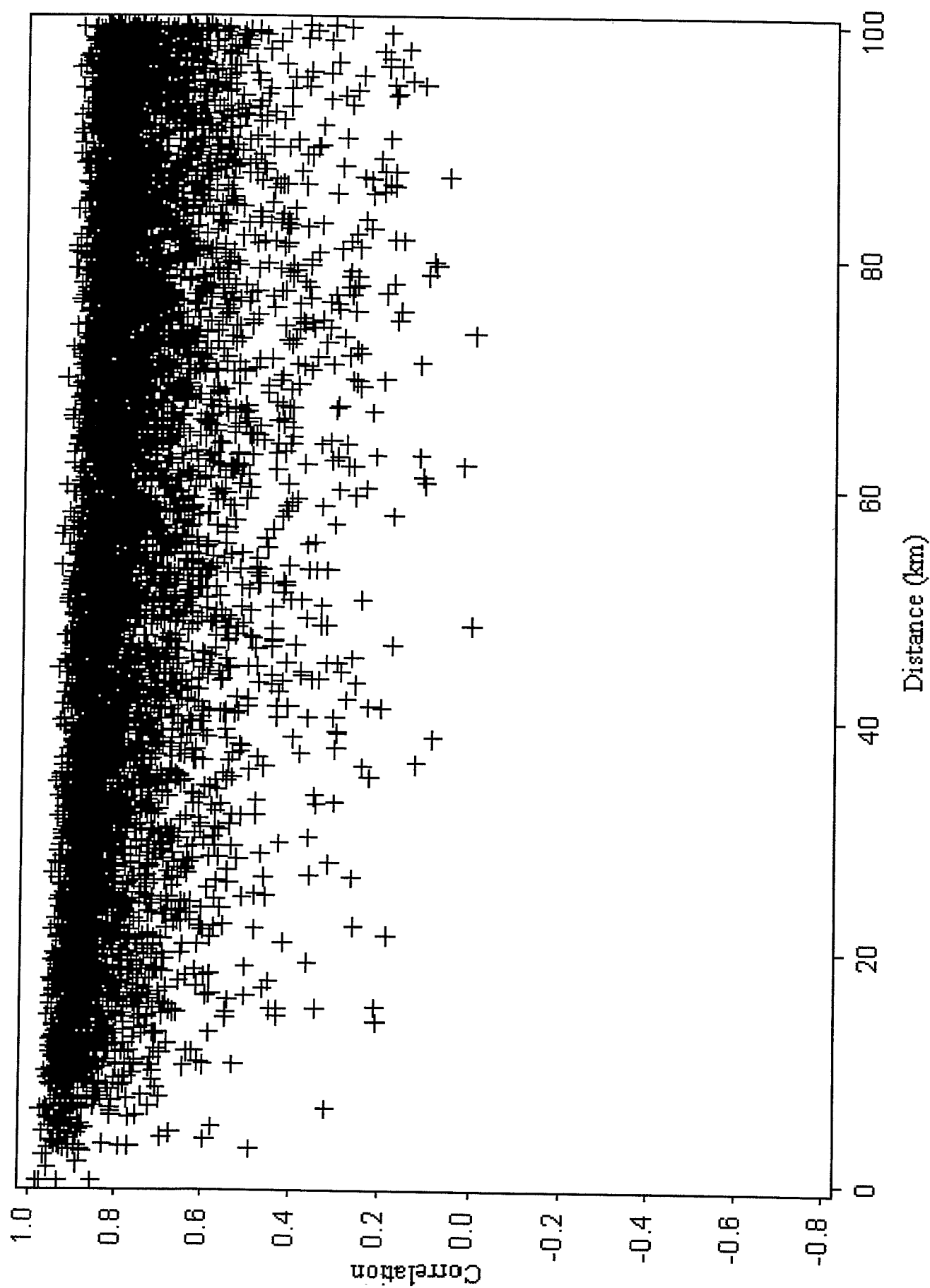


Figure F-3. Ozone Hourly Correlations vs Distance

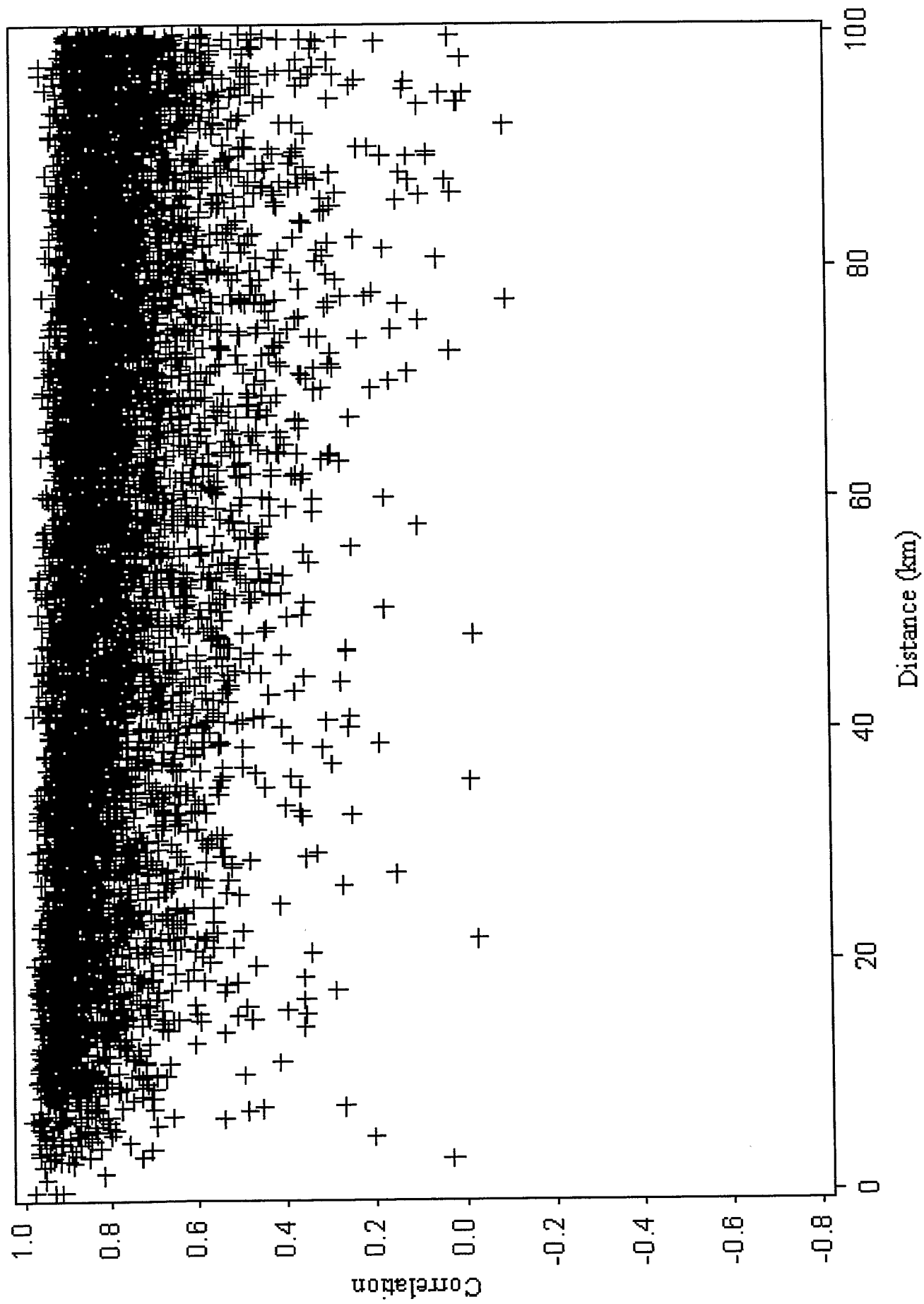


Figure F-4. Ozone Daily mean Correlations vs. Distance

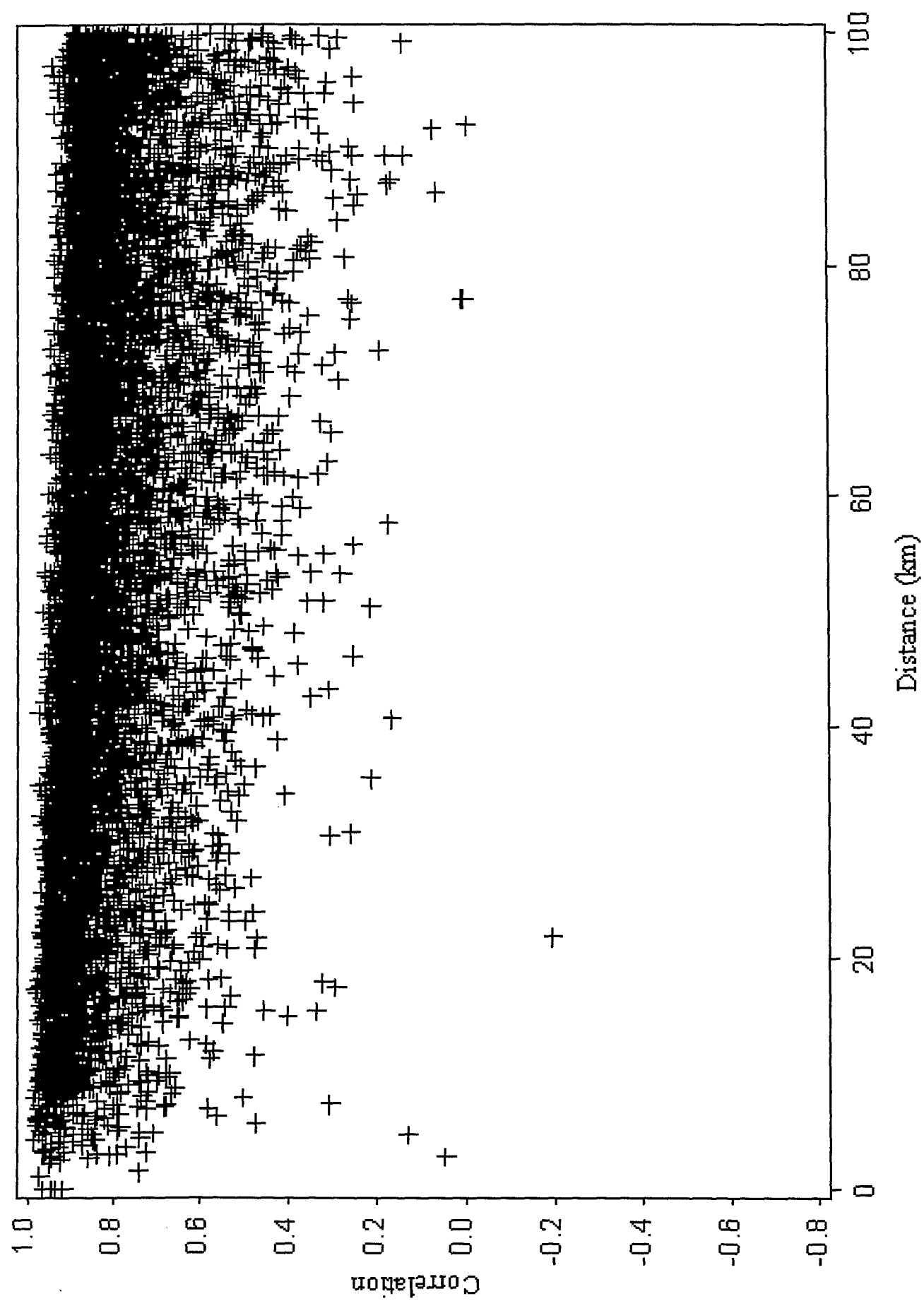


Figure F-5. Ozone Daily max Correlations vs. Distance

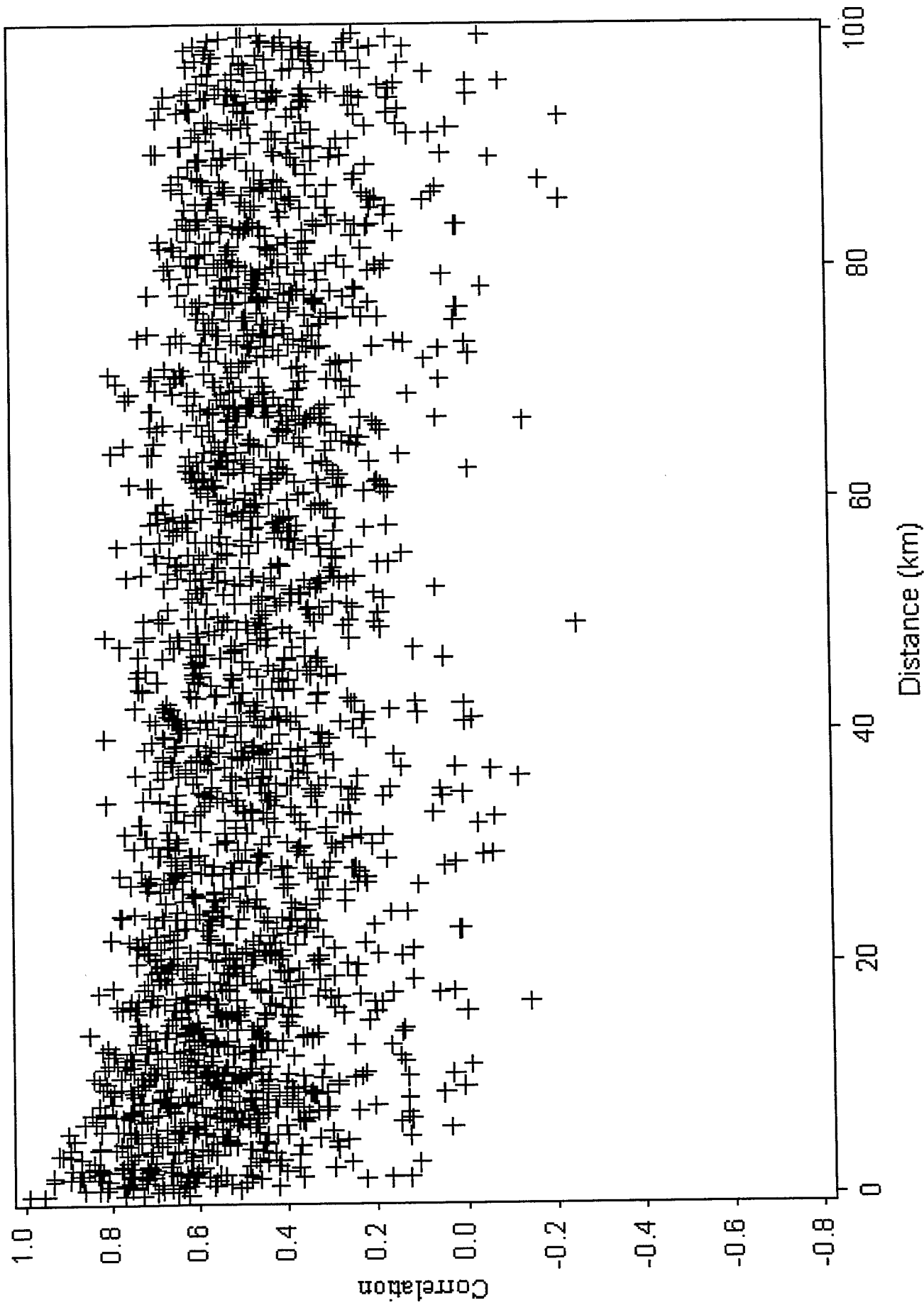


Figure F-6. 00 HourlyCorrelations vs. Distance

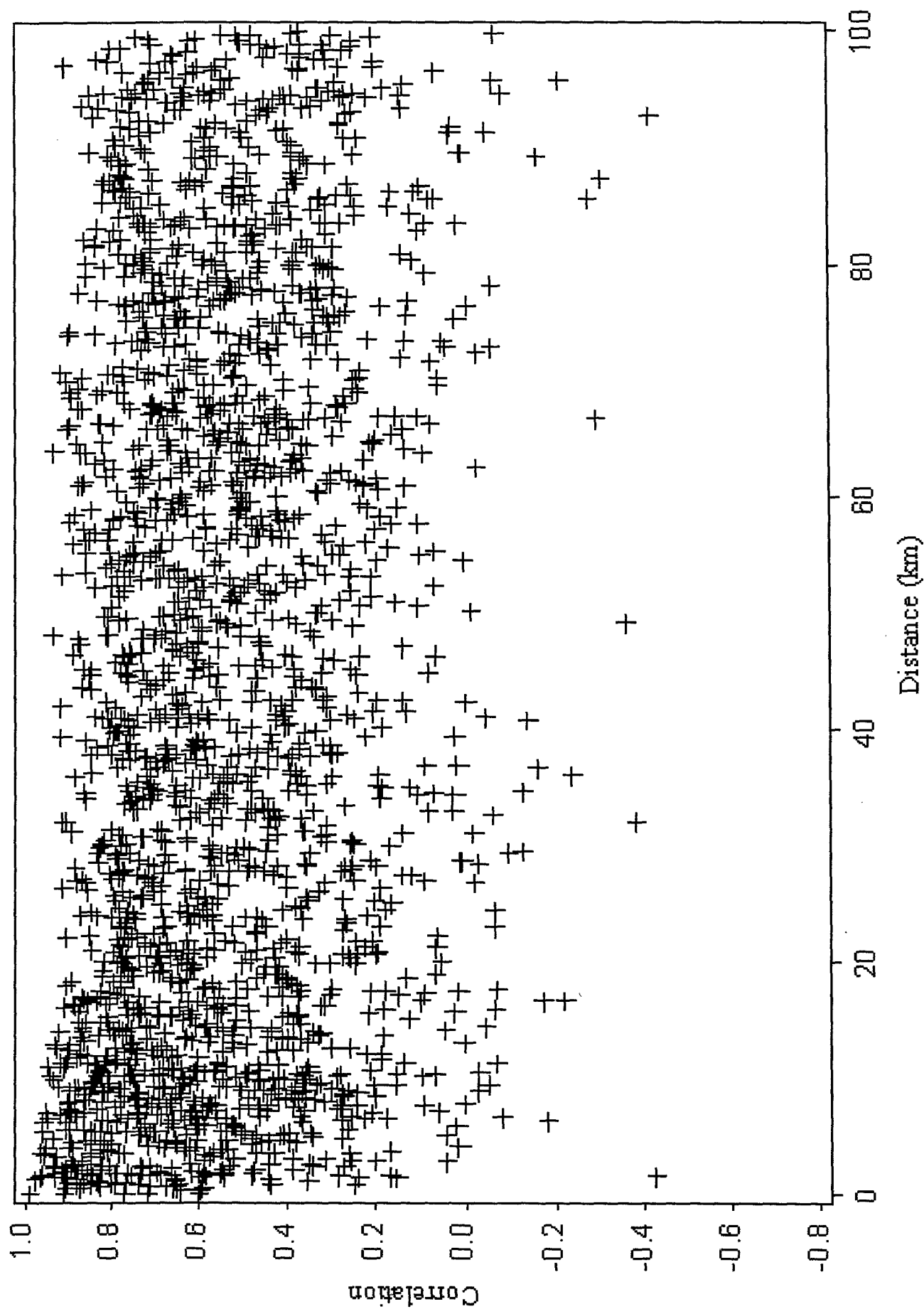


Figure F-7. ∞ Daily mean Correlations vs. Distance

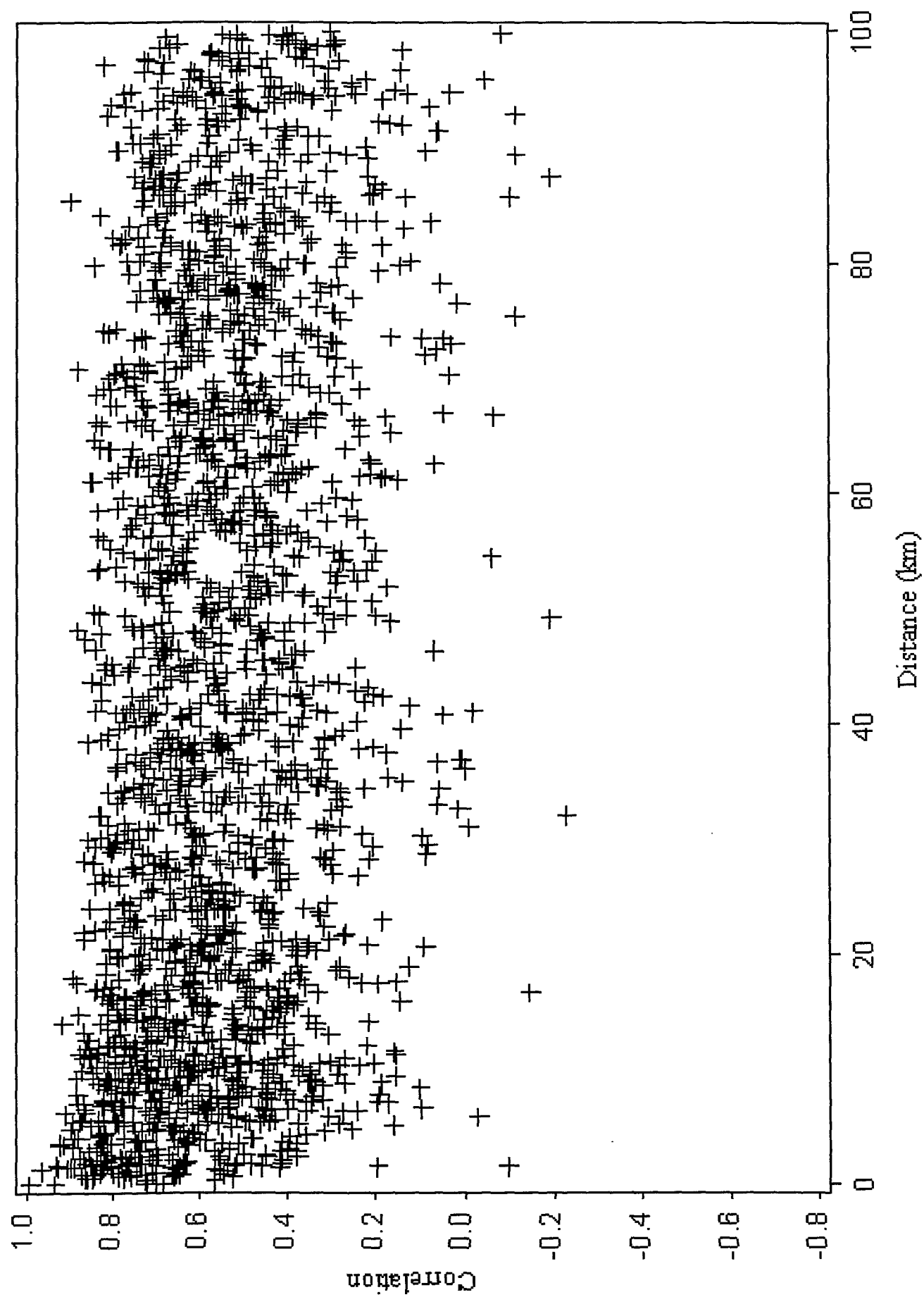


Figure F-8. CO Dailymax Correlations vs Distance

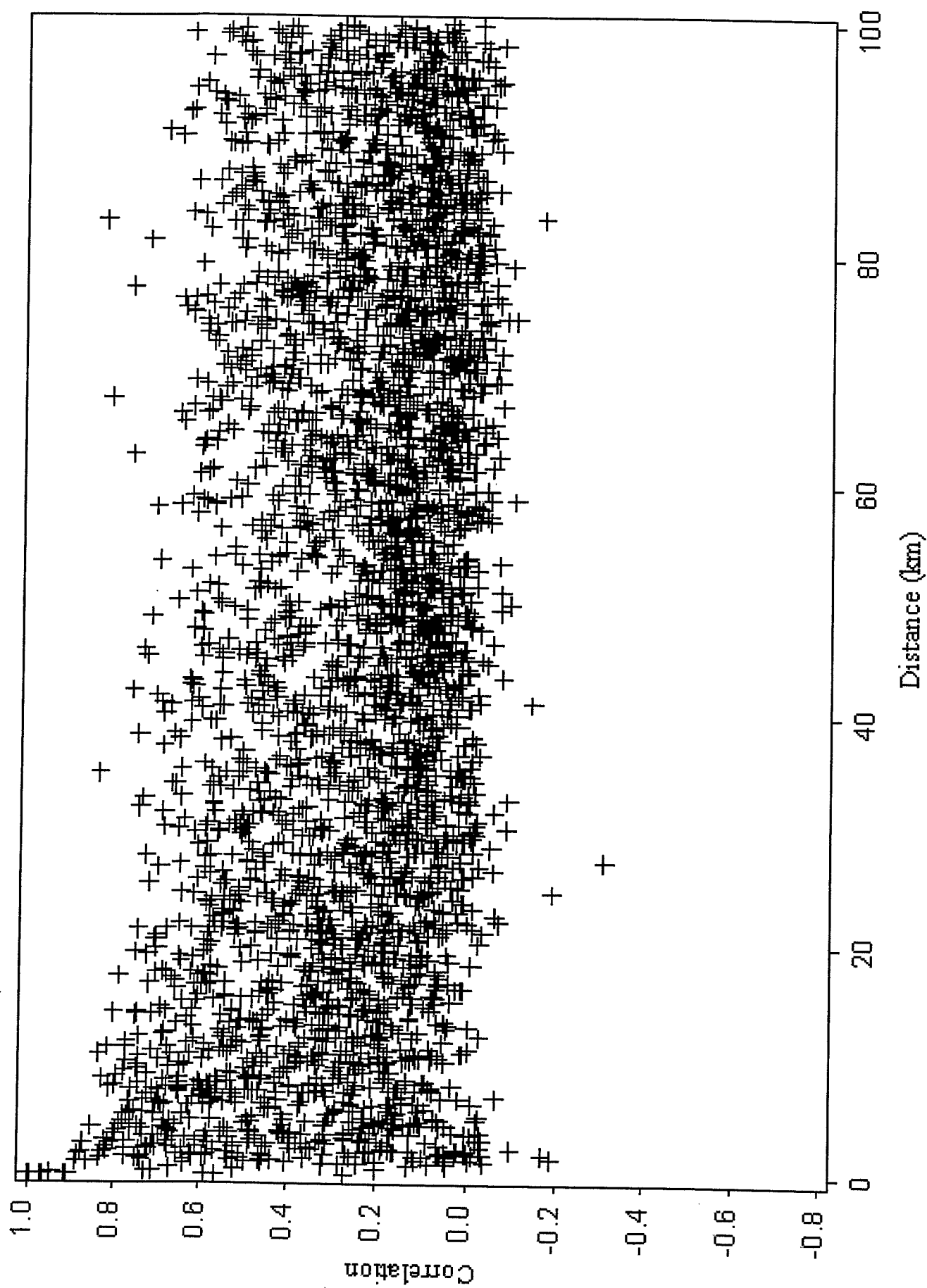


Figure F-9. SO₄ Hourly Correlations vs. Distance

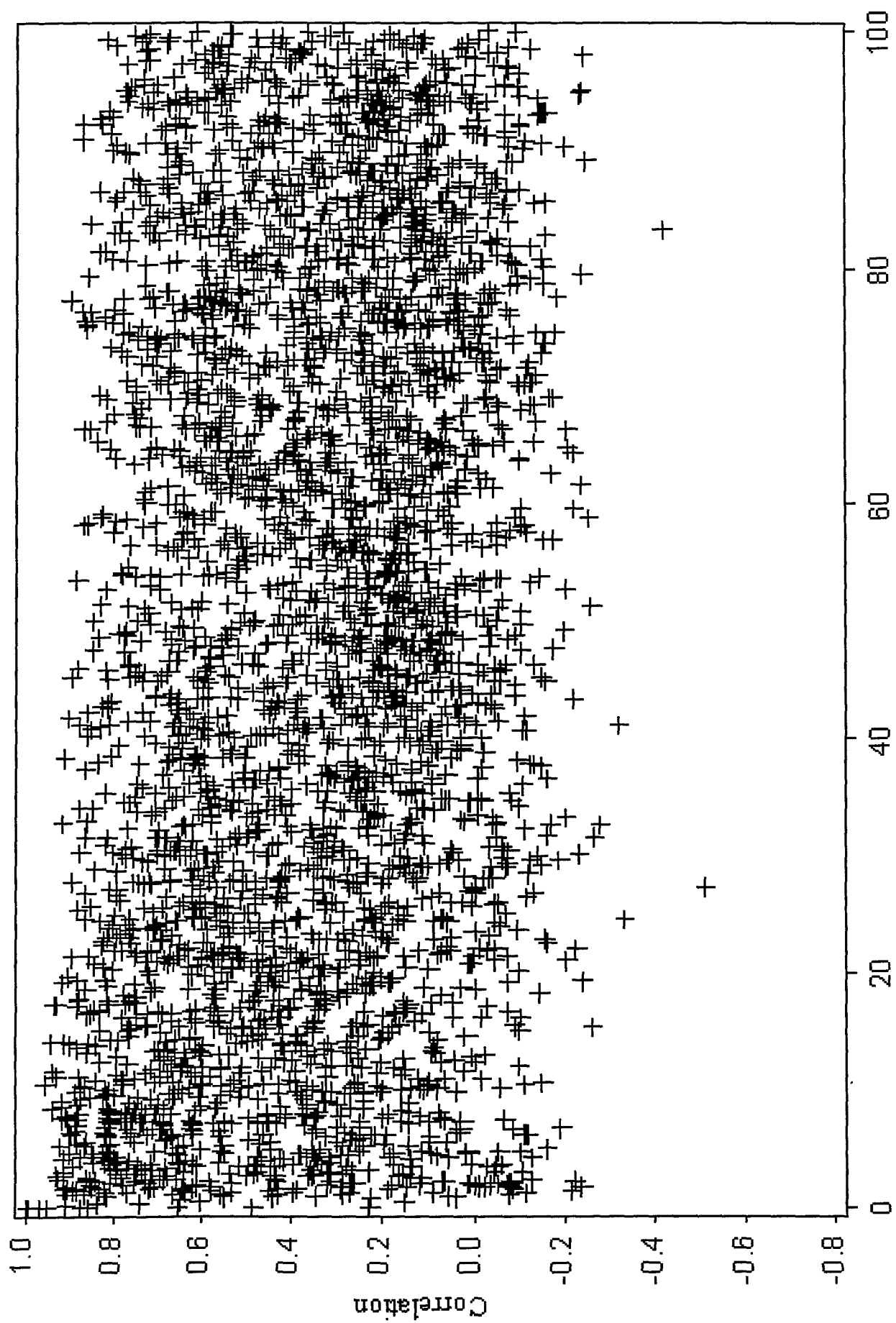


Figure F-10. SO₂ Daily mean Correlations vs. Distance

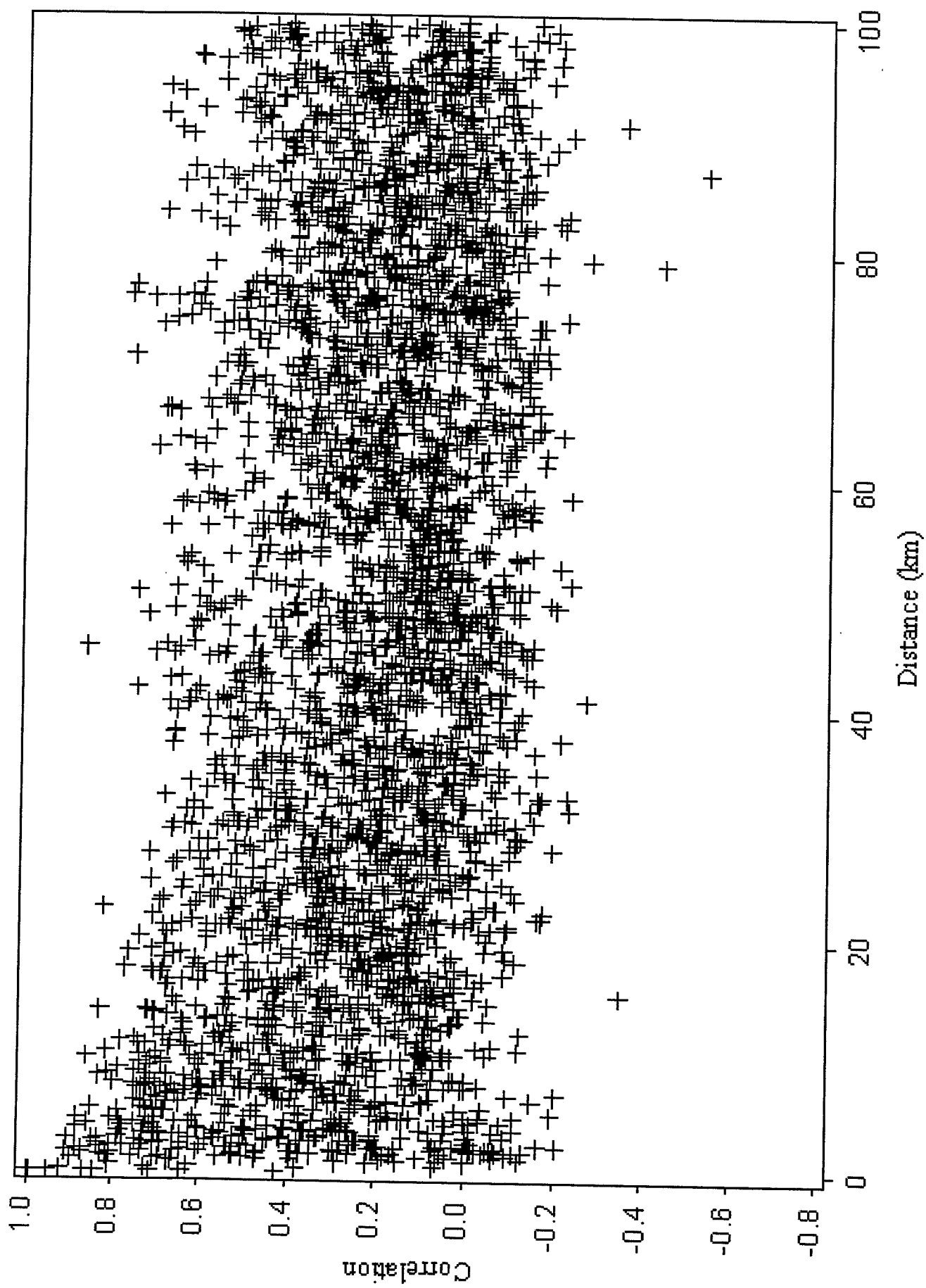


Figure F-11. SO₂ Daily max Correlations vs. Distance

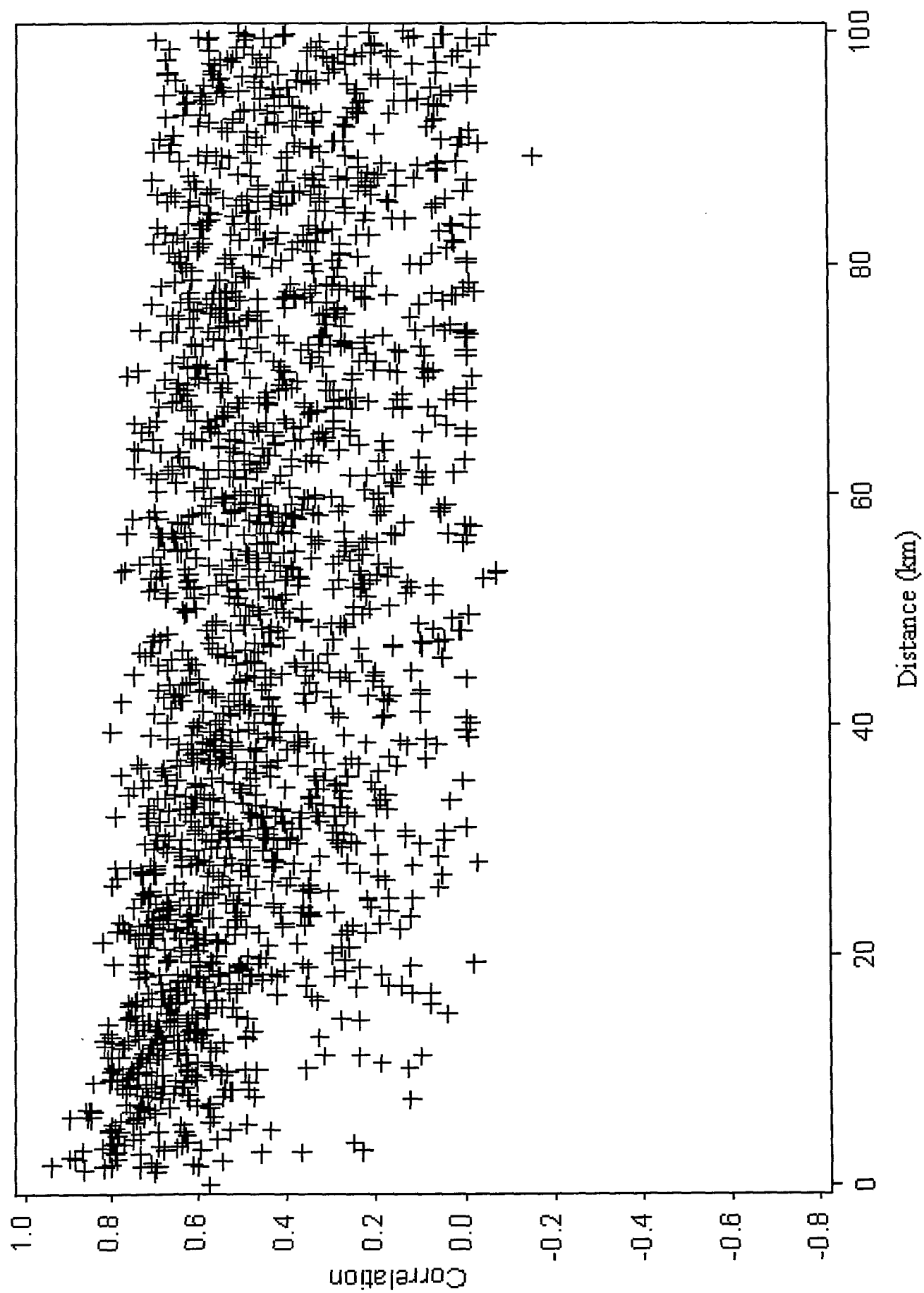


Figure F-12. NO_x Hourly Correlations vs Distance

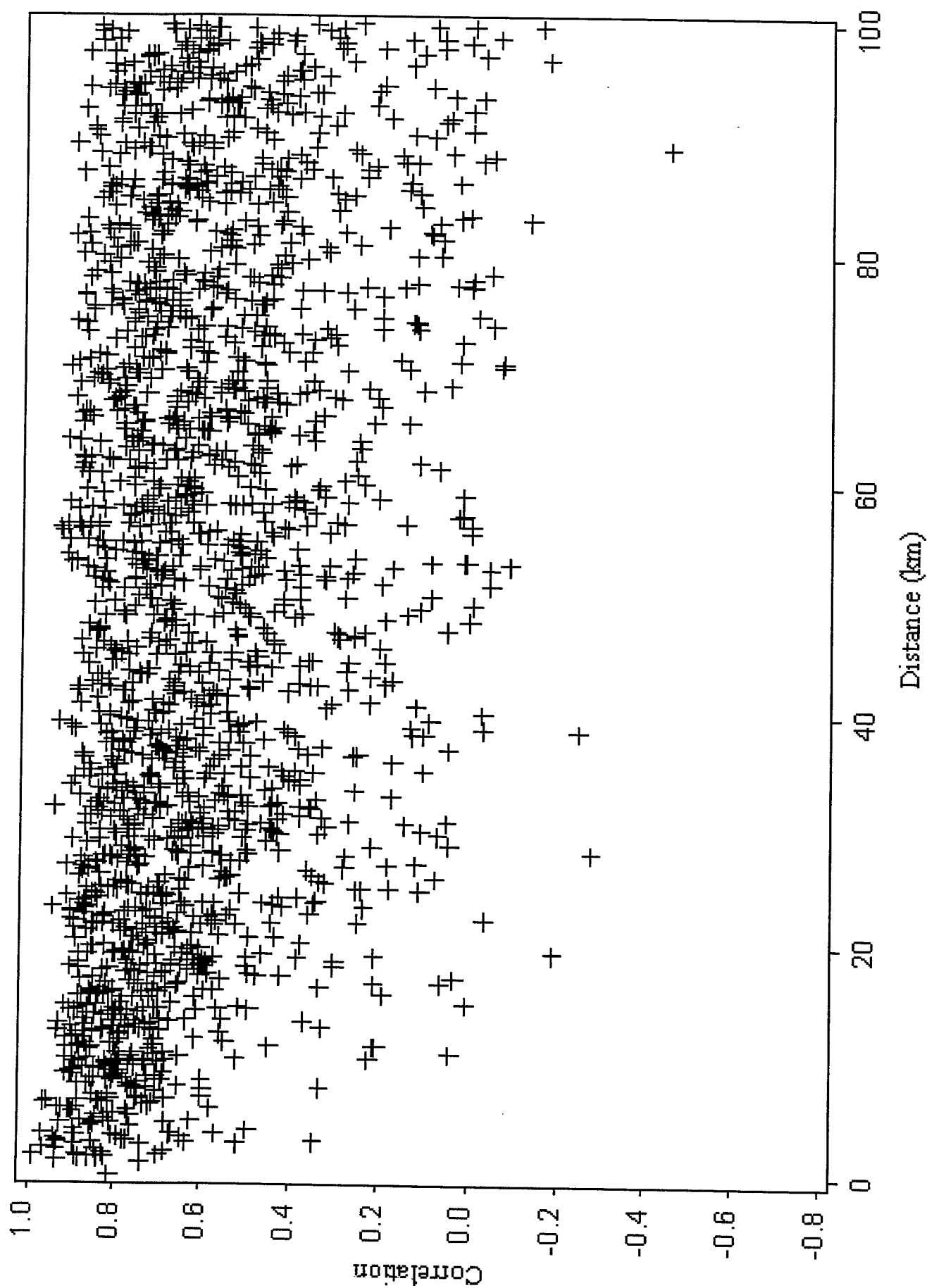


Figure F-13. NO_x Daily mean Correlations vs distance

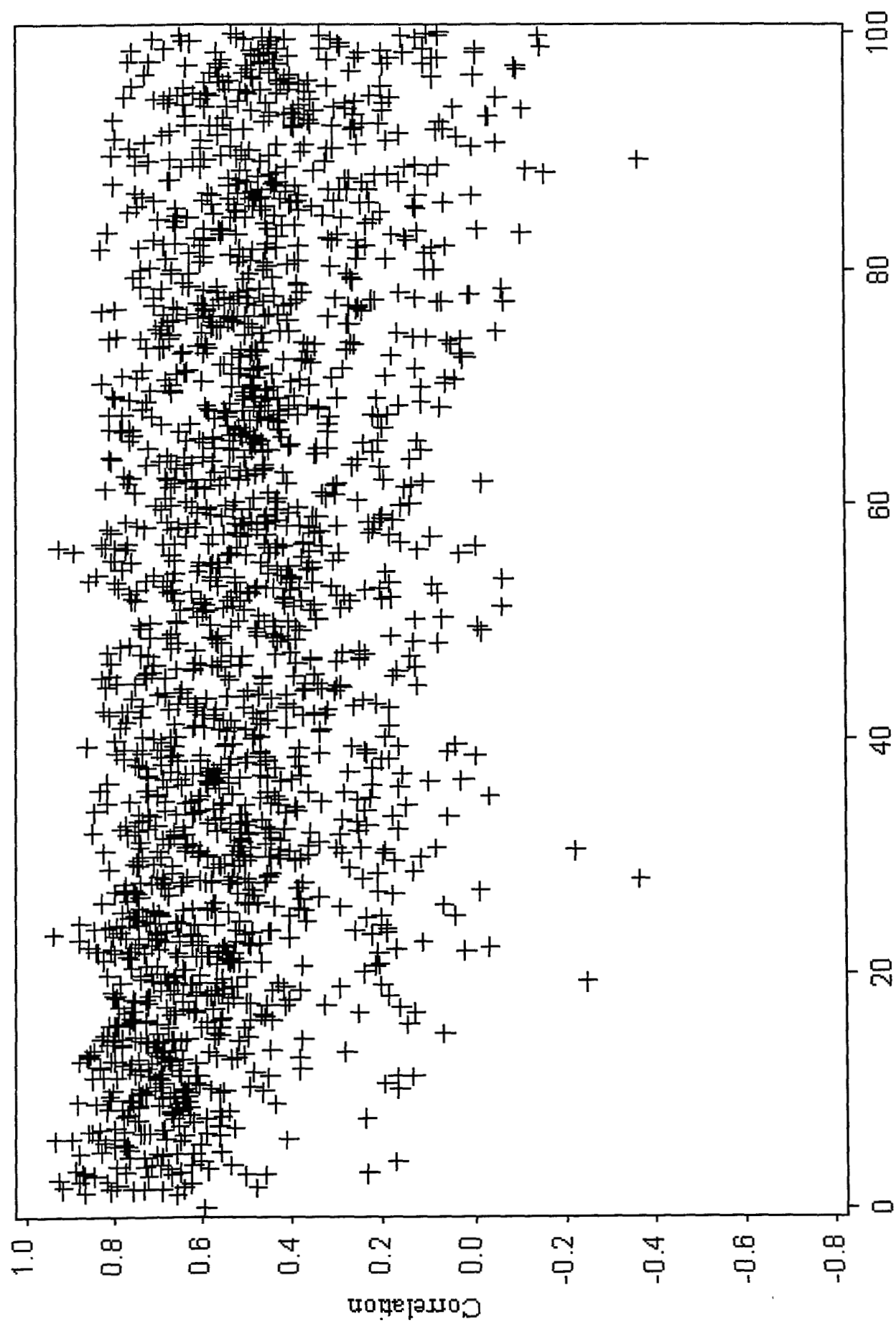


Figure F-14. NO₃ Daily max Correlations vs Distance